parrhesia 23 · 2016 · 129-154

axiomatic set theory in the work of deleuze and guattari: a critique

jon roffe

Over the last decade, or more precisely since the translation of Alain Badiou's *Being and Event* in 2005, the significance of mathematics in the trajectory of twentieth century French thought has become increasingly perspicuous. Indeed, Badiou increasingly appears to as the recipient of a rich but often overlooked tradition of philosophers who engaged with mathematical problematics.[1]

To make this observation is also to note that an adequate familiarity with certain strains of French thought make the rather steep demand of a concomitant familiarity with mathematics. This is true even in cases where the mathematical references might appear at first glance to be somewhat slight. The paradigmatic case in this regard is the work of Gilles Deleuze. While his references to various branches of mathematics—calculus, group theory, model theory, analysis, and differential geometry for instance—may appear to be fleeting, they constitute important and sometimes even indispensable points of reference.[2]

The aim of this piece is to consider a branch of mathematics that is treated even more fleetingly than many others by Deleuze, the one which Badiou's work has done so much to focus attention on: axiomatic set theory.

Deleuze and Guattari's *What is Philosophy?* includes two short invocations of set theory. The first appears in the chapter devoted to science, defined as the act of thought that organizes and measures states of affairs. This capacity itself turns around the creation—science being an act of creation in their view, equal if different to philosophy and art—of the limits and independent variables that compose scientific functions. These limits are of necessity engendered in relationship to the movement of infinite speed that characterizes what they call chaos, that is, being as such. Given this situation, Deleuze and Guattari note, one can ask how it is that a created limit (such as zero, the speed of light or zero Kelvin) can gain a foodhold in what seems to absolutely exceed it: "It is difficult to see how the limit immediately cuts into the infinite, the unlimited."[3] A first answer comes in the form of a brief presentation of Cantorian set theory:

> Cantor provides this theory [according to which the determination instantiated by a limit immediately relates to the infinite] with its mathematical formulas from a double—intrinsic and extrinsic—point of view. According to the first, a set is said to be infinite if it presents a term-by-term correspondence with one of its parts or subsets, the set and the subset having the same power or the same number of elements that can be designated by 'aleph-0', as with the set of whole numbers. According to the second determination, the set of subsets of a given set is necessarily larger than the original set: the set of the parts of aleph-0 therefore refers to a different transfinite number, aleph-1, which possesses the power of the continuum or corresponds to the set of real numbers. (WP 120, translation modified)

Contracted here is an account of certain of Cantor's famous contributions to mathematics. First, he shows that we need not conceive of the infinite in terms of the Aristotelian *apeiron*. Instead, we can take as a whole collection—a set—an infinite series like the series of natural numbers (1, 2, 3 ...). This set, written as $\mathbb{N}$, is a first-order infinite set, but not the only one. It can be demonstrated that the set of all even numbers, the set of all integers $\mathbb{Z}$, the set of all rational numbers $\mathbb{Q}$, the set of all prime numbers, and any set produced by adding to or multiplying any of these sets (for example, $\mathbb{N}$+100 or 2$\mathbb{Q}$) are the same size. These sets are said to be equinumerous, having the same *power*, or possessing the same *cardinality*.

Cantor proves these *prima facie* counter-intuitive claims by way of his famous method of diagonalisation, a *reductio ad absurdum* that demonstrates that there are sets which are not included in $\mathbb{N}$ and thus exceed it in size. Principal among these is the set of real numbers, $\mathbb{R}$. Most importantly, it is also possible to define a set on the basis of ℕ that exceeds its size. This set, called the power set of ℕ or P(ℕ), is the set of all of the subsets of ℕ, whose size is 2^n, where n is the number of members that belong to ℕ.[4] Moreover, the same argument also shows that ℝ and any sets of the same or higher cardinality are uncountable or non-denumerable in nature—essentially due to the fact that P(ℕ) has the same cardinality as ℝ—a result that is now known as Cantor's theorem. That is, a set is non-denumerable if a) it is infinite, and b) there is no bijection between the set and ℕ. We will return to this point in what follows.

The final element of Cantor's work that Deleuze and Guattari invoke in this passage is the continuum, which to this day marks an unresolved issue in set theory. Famous results due to Kurt Gödel on the one hand and Paul Cohen on the other have shown that the continuum hypothesis, as it is known, is independent of axiomatic set theory—that is, it can neither be proven nor disproven on the basis of the axioms of ZFC. This hypothesis, which Cantor spent the last part of his life trying in vain to prove, asserts that there is no set whose size falls between those of ℕ and ℝ, א-0 (aleph zero)and א-1.

Cantor's failure to prove the continuum hypothesis was only the first critical development, however, since, when thinkers like Russell and Frege attempted to formalise the foundations of mathematics in versions of first-order predicate logic, they discovered a number of crippling paradoxes. These paradoxes are numerous, and include the paradox of self-belonging—which, in Frege's own words, "the sole possible foundation of arithmetic,"[5] on logical grounds—the Burali-Forti paradox of the largest ordinal, and that of the largest cardinal.[6] Various forms of axiomatic systems were developed—the Zermelo-Frankel (ZF) axiomatic being only the most well-known—developed in order to address these paradoxes and Cantor's inability to prove CH.

Given that Badiou's use of set theory is the object of the note in *What is Philosophy?*, it is clear that they are making reference to ZF set theory in particular, which has, on the now standard presentation, ten axioms.[7] For the most part, the axioms are operational, defining what one is capable of doing on the basis of a given set (for example, the Power Set axiom that we have already seen). However, there are also two existential axioms, axioms that assert the existence of a certain set: the axiom of Infinity, which asserts the existence of the infinite set constituted by ℕ; and the axiom of the null-set, which asserts the existence of a unique set with no members. Finally, there is axiom of Foundation (or Regularity), which rules out sets that belong to themselves, and, more generally, allows for the ordered ranking of sets.

The effect of this axiomatisation of Cantorian set theory was to render it remarkably (if not provably) coherent. Even in light of the famous Gödel results, which show that there is no way to absolutely set theory on its own terms, it is widely taken to be the most fundamental branch of mathematics, and the one in which most others can be formulated.

While the above otiose summary of certain set theory basics may not yet seem particularly interesting or relevant, this final point is obviously significant, for it exactly contradicts what Deleuze and Guattari state at the end of the citation above. We cannot say of aleph-1 that it possesses the power of the continuum, a claim that also conceals a second error and that we will return to below. This is a strange error of presentation to make, made stranger yet again by the fact that they invoke, in the context of their second reference to set theory in a short text on Badiou (WP 150), precisely the excessive character of the power set that makes its location with respect to the hierarchy of alephs problematic.[8]

There are further problems too. By equating the infinite with both speed and the figure of the unlimited, Deleuze and Guattari's account sets them at odds with set theory. On the one hand, sets are extensive multiplicities, entirely static in character and thus incommensurable with any dynamist interpretation of the infinite, and here the development of set theory of a piece with the more general trend in 19th century mathematics to evict dynamism that Deleuze remarks in *Difference and Repetition* in the course of his presentation of differential calculus.[9] On the other, and while there may be other ways of defining the infinite, there is no way to reconcile the notion of *apeiron* with the set theoretic conception of the infinite, and it is the latter that is manifestly at play in this citation but also throughout *What is Philosophy?*

■

What makes this mismatch between set theory and Deleuze and Guattari's presentation of it particularly troubling is the fact that a substantial reference to set theory forms the lynchpin of their account of capitalism as a social formation.[10] If, that is, their analysis of set theory is fallacious, unfortunate consequences may accrue on this front also. This latter is not quite the object of what follows, however. My aim instead is to trace the fault-line in Deleuze and Guattari's understanding of set theory that leads to these passages in *What is Philosophy?* While the context of the most detailed discussion of set theory is their account of the capitalist social formation, there is no need to insist that set theoretic mathematics and capitalism share *literally* the same object. This assertion however undergirds Deleuze and Guattari's argument. It is enough to traverse the serial treatment of set theory in *A Thousand Plateaus* to demonstrate that the analysis of capitalism is poorly founded, and this is what the following argument is meant to demonstrate. However, this does not lead to the conclusion that the analysis of capitalism through the concept of the axiom is entirely misplaced. In the final section of the article, I will present a more modest way of framing this concept that does not require the falsification of set theory.

After briefly outlining the account of capitalism as an axiomatic presented in *Anti-Oedipus* and *A Thousand Plateaus* and some of its examples, we will consider the use they make of Robert Blanché's *L'axiomatique*, the most significant secondary reference deployed by Deleuze and Guattari in accounting for what they call the capitalist axiomatic in *A Thousand Plateaus*. Third, we will critically consider the "summary sketch"[11] of the axiomatic mode of thought for the analysis of capitalism that closes the thirteenth plateau of the same text, considering it from the point of view of its fidelity to axiomatic set theory. Again, let me emphasise that this effort, technical though it may appear, is essential if we are to properly take the measure of one of the most frequently espoused elements of Deleuze and Guattari's thought: their politics.

WHAT IS AN AXIOM FOR DELEUZE AND GUATTARI?

Deleuze and Guattari define capitalism as "the only social machine that is constructed on the basis of decoded flows, substituting for intrinsic codes an axiomatic of abstract quantities,"[12] in which nation-States function no longer as overarching structures but diverse models of realization. That is, in the capitalist formation, the absolute role of qualitative hierarchies gives way to an ensemble of mechanisms that treat social reality on quantitative terms, which Deleuze and Guattari call axioms.

The most direct definition of the term 'axiom' that Deleuze and Guattari give is found in *A Thousand Plateaus*. Axioms, they say, are rules that deal "directly with purely functional elements and relations whose nature is not specified, and which are immediately realized in highly varied domains simultaneously." (TP 454) An axiom is thus a rule indifferent to the nature of what it is applied to and to the context of its application. They immediately oppose this to the kinds of rules, namely *codes*, that characterize other social formations: "codes, on the other hand, are relative to those domains and express specific relations between qualified elements that cannot be subsumed by a higher formal unity (overcoding) except by transcendence and in an indirect fashion." (TP 454) By extension, then, to mark the indifference of axioms in this way is to mark their absolute neutrality with respect to qualitative or evaluative positions. Nor, we should note, are they strictly speaking *mediated* by qualitative concerns (a point that we will return to below). It is worth insisting too that for Deleuze and Guattari, axioms are never found in isolation. Even in the case of totalitarian States, they will insist that at least three sets of axioms are to be found. This is one reason, though not the most consequential as we will see, why they tend to speak of the capitalist axiomatic rather than discrete sets of axioms. If we compare this to the brief sketch of ZFC provided above, it is clear at the two approaches are indeed perfectly compatible in general terms.

Before proceeding, it is important to locate the role of the State in capitalism, for it is not the case, in Deleuze and Guattari's view, that this mode of social organization is entirely dissolved upon the advent of capitalism. Instead—and this is a claim we will return to in what follows—the State takes on the role of "model of realization" for the capitalist axiomatic itself. By this, they mean that the State, no longer ultimate locus of political sovereignty, functions as a pluralized set of regulatory apparatuses, effectively assuring that the axioms have the appropriate material and expressive matters required for their functioning—infrastructure and material requirements, but also elements like appropriate legal frameworks and political alliances. It is therefore *through* States that capitalism is realized—not *in accordance with* State regulation but *by way of it*.

In *A Thousand Plateaus*, Deleuze and Guattari will provide a tripartite typology of States conceived in this way. Summarily speaking, they suggest that all States as models of realization of the capitalist axiomatic can be contrasted in terms of 1) the number of axioms that are deployed therein, 2) the respective relations of production (in Marx's sense of the term), and 3) the forms in which States co-opt peripheral economic systems in the service of the realization of the axioms currently in play. Together, these three axes allow for the construction of a phase-space of global capitalism, in which every concrete State apparatus can be located.[13]

BLANCHÉ'S *L'AXIOMATIQUE*

This summary provides a general sense of the stakes of the concepts of axiom and axiomatic from the point of view of Deleuze and Guattari's account. Behind the presentation they give, there is, as is so often the case in their work, a broad set of references, among them Marx, Clastres, Childe, Amin, Braudel, Lévi-Strauss, and Dumézil. However, concerning the mathematical provenance of the concept of the axiom, we find in all of Deleuze and Guattari's texts on the matter only a single touchstone, Robert Blanché's 1955 *L'axiomatique*.[14] To be more specific, and while they marshal a number of proper names around their invocation of axioms, including a roll-call of intuitionist mathematicians, only Blanché's text is deployed directly (the single exception to this case, concerning the included middle, will be examined in what follows). Moreover, the invocation of intuitionism[15] does not bear on the use that Deleuze and Guattari make of axiomatics, but only the need to mark a point beyond which the framework of axiomatics is no longer adequate—beyond the capitalist axiomatic, there is the calculus of problems involved in the creation of revolutionary connections, a claim with which Deleuze and Guattari close the "Apparatus of Capture" plateau.

Blanché's text is devoted, not to set theory as such, but to the axiomatic method more generally—from its beginnings in Euclid through to post-Gödelian mathematical logic—and also considers the significance of the axiomatic method in science and its consequences for philosophy. He is particularly concerned to clarify the limits of the method as they appear, on the one hand, in relation to the irreducible intuitive commitments of the method: "it is only in books that that an axiomatic begins with axioms." (A 87) On the other, he insists on the kinds of paradoxes (notably Skolem's paradox, to which we will return below) that seem to show the inevitable relativity of axiomatic systems, and more generally that there are metamathematical questions that are unable to be resolved by the axiomatic method on its own terms. But Blanché's key contention is the obverse and complement of this point, namely that the axiomatic method allows for a regulated rapprochement of the formal and the empirical, a broaching of "the old distinction between rational and empirical science." (A 103) It does this precisely by providing a means to systematically formalize the role of the basic experiential elements of mathematical science, like the 'scribble' that makes up numbers on a piece of paper or blackboard.

Deleuze and Guattari make use of the parts of Blanché's text that address certain central problematics in the axiomatic method itself, those which arise primarily (if not exclusively) in the axiomatisation of set theory. To these problematics, Deleuze and Guattari devote a numbered list that closes the argument of "Apparatus of Capture" and constitutes an indispensable part of their account of capitalism in *A Thousand Plateaus*. They frame this treatment with the following remark:

> Our use of the word 'axiomatic' is far from a metaphor; we find literally the same theoretical problems that are posed by the models in an axiomatic repeated in relation to the State [...] These 'problems' become singularly political when we think of modern States. (TP 455)

Later they will add that "It is the real characteristics of axiomatics that lead us to say that capitalism and present-day politics are an axiomatic in the literal sense." (TP 461) This is key. Whatever Deleuze and Guattari hope to accomplish by deploying axiomatics, it will not be of the order of a metaphorical gloss. Second, the value of the axiomatic method will be with respect to the relationship between the State and capitalism. Third, it is by treating problems of the axiomatic method in this setting that they reveal their political character. We set aside here the third point, and focus on the first—that is, the key question will be whether or not the elaboration of these problematics of the axiomatic method presented by Deleuze and Guattari is literal in character. The answer can be found by treating each of the seven problematics that they discuss in turn.

Before doing so, we need to consider the second point, since what is fundamentally at issue in Deleuze and Guattari's treatment is less axiom-systems on their own terms than the relationship between a set of axioms and the various models of realization through which they may be deployed. In a nutshell, this second claim amounts to the assertion that the pair axiomatic/model in the mathematical deployment of set theory is the means by which we can think the pair axiomatic/State.

Rather than considering ZF, let's take a simpler case, that of Peano arithmetic (PA).[16] PA is a set of axioms that define the natural numbers, and the basic arithmetical operations of addition, subtraction, multiplication, division, and comparison by size (that is, linear ordering). It contains nine axioms, of which five are basic,[17] and can be paraphrased as follows:

1) *Zero* is a natural number
2) The *successor* of a natural number is a natural number
3) No two natural numbers have the same successor
4) Zero is not a successor
5) If a property belongs to zero, and if, when this property belongs to a number it also belongs to its successor, it belongs to all numbers

Which numbers, though, do these axioms pertain to? The answer is that there are an infinite number of *models* of PA. It is important to see that model here does not mean a prior ideal structure, and "should not suggest the idea of an archetypal anteriority." (A 46n1) Rather, a set of axioms is modeled in the way that a mold is filled with plaster; a model of an axiomatic system is some ensemble of propositions, groups, or sets—in a word, various structures—for which all of the claims of the axiomatic system hold. One such model of PA is clearly $\mathbb{N}$, the set of all natural numbers. However, the set of all even numbers is also a model, since all of the axioms hold for this set as well, since the successor relationship does not rely upon any prior ordinal series. The same holds for the set of numbers beginning with 100, since the first axiom above can take this as zero. Thus, while models vary, they are models *of* an axiom set to the extent that they are various expressions of the axioms it includes. At the limit, the axiomatic method turns on the capacity to show that, for the entire range of possible models, there is a single axiom set to which they correspond. In Blanché's words, "The axiomatic method is interested precisely in revealing isomorphisms between apparently heterogenous concrete theories, by referring them back to the unity of an abstract system." (A 46)

ADDITION, SUBTRACTION AND SATURATION OF AXIOMS

With this example in hand, we can turn to the seven points that characterize the capitalist axiomatic and its realization. The first two revolve around the issue of saturation.

Deleuze and Guattari begin by considering the very variable forms of the State under capitalism, with respect to the number of axioms deployed. What happens though in the two limit cases? On the one hand, can there be a capitalist State that lacks axioms altogether? On the other, can there be such a State in which no further axioms can be added? In the first case, the answer for Deleuze and Guattari is—both in principle and in fact—no. No State may exist in capitalism without axioms, since to be such a State is simply to be a model of realization of the capitalist axiomatic. They consider totalitarian and fascist States as the extreme cases here. In both, there is a "tendency to restrict the number of axioms," (TP 462) but without eradicating them altogether. Furthermore, in the case of fascism, the restriction is doubled by what they call a "tautological or fictitious proliferation [of axioms], a multiplication by subtraction," (TP 463) that arises because of the war economy embodied by such States. The term in mathematics to describe this situation is apt—we are looking at a *weak* axiomatic system, not an absence of such a system. (A 53-4)

What of the second case, the possibility of saturation? With respect to set theory, a saturated model is one to which no further independent axiom can be brought to bear—it is a model whose entire structure is accounted for by the axioms already in play. For Deleuze and Guattari, however, saturation is not possible.

On the one hand, the impossibility of saturation arises because of what they call the fundamental law of capitalism, expounded for them above all by Marx in book three of *Capital*: "Capitalist production seeks continually to overcome these immanent barriers, but overcomes them only by means which again place these barriers in its way and on a more formidable scale."[18] In Deleuze and Guattari's words, "capitalism confronts its own limits and simultaneously displaces them, setting them down again farther along." (TP 463) Consequently, there is no opportunity for the situation of saturation to arise, since the most general level of the capitalist system is dynamic rather than static. As they write in *Anti-Oedipus*: "How much flexibility there is in the axiomatic of capitalism, always ready to widen its own limits so as to add a new axiom to a previously saturated system!"[19]

On the other hand, the criterion of contradiction has no purchase on the ensemble of axioms deployed in capitalism. Indeed, on Deleuze and Guattari's view, this is part of the very flexibility of the capitalist system itself. To once again cite the searing remark from *What is Philosophy?*: "Human rights are axioms. They can coexist on the market with many other axioms, notably those concerning the security of property, which are unaware of or suspend them even more than they contradict them."[20] This is reminiscent of a claim central to psychoanalysis, viz., that there is no negation or contradiction in the unconscious, the drives being subordinate to no logical exigency. The same holds for the axioms in capitalism: contradiction is strictly irrelevant, given that their (meaningless) functioning as vectors of price is all that is at stake.

Already in these first two claims we see a rift appearing between Deleuze and Guattari's use of the notion of the axiom and that which is found within mathematics, a rift that grows as the sequence of points continues.

PRIORITY AND INDEPENDENCE

Deleuze and Guattari's third point concerns the variety of States, no longer from the point of view of the axiomatic, as above, but from the point of view of the States as models of realization. Here, since they make what seems to be a correct, indeed strictly literal, use of the mathematical framework, we can be brief.

We saw earlier that States qua models of realization of the capitalist axiomatic vary along three axes: isomorphic heterogeneity, heteromorphy and polymorphy. Regarding the isomorphy of States, they write that

> The general rules of this are as follows: the consistency, *totality* [l'ensemble] *or unity of the axiomatic* is defined by capital as a 'right' or a relation of production (for the market); *the respective independence of the axioms* in no way contradicts this totality but derives from the divisions or sectors of the capitalist mode of production; *the isomorphy of the models*, with the two poles of addition and subtraction, depends on how the domestic and foreign markets are distributed in each case. (TP 464)

Clearly, this is an elaboration of the trivalent typology of States. State heteromorphy can be conceived in terms of the differing deployment of a coherent set of axioms that constitute the States in question at a given moment. By putting it in the way they do, Deleuze and Guattari wish to emphasise what they consider to be the secondary role of Marx's concept of mode of production in capitalism. Modes of production are, on their view, expressions of a given set of axioms in a capitalist State, and thus subordinate to the axiomatic in a more general sense.[21]

In turn, the concept of State polymorphy relies upon the independence of the various axioms. In its mathematical deployment, independence describes a relationship between axioms such that no one can be derived from any other. There is, at the level of the axioms that make up an axiomatic system, no relation of dependence. Deleuze and Guattari use this formulation in their analysis of capitalism in order to note that this State polymorphy involves enrolling other States, even those whose relations of production are non-capitalist, in the realization of the capitalist axiomatic. Given this, it is clearly the case that a different subset of axioms will be required in these satellite or second-order States—the axioms that govern the market in high end sports shoes will necessarily differ from those deployed in the third-world countries where the shoes are manufactured.

But now a further problem now arises around the notion of saturation. We can characterize States as models of realization of the capitalist axiomatic by deploying the notions of totality, independence and isomorphy, but only insofar as we are speaking about a single axiomatic variously modeled. But the first two points (concerning addition, subtraction and saturation) are framed in such a way that we are speaking not of one axiom system but a plurality. In other words, what we have seen so far is an insistence on a plurality of axiom-sets and a plurality of models of axiom-sets. The State is what specifies the latter, which is to say that States are themselves these plural models of realization. But what specifies the former? The question then seems to be the following: what is it that divides up the system of axiomatic conjunctions that, in a general sense, constitutes global capitalism? To say it is the States seems to give up the emphasis on capitalism as being the general determinant of contemporary social organization.

At the same time, it seems difficult to see what kind of agency we might invoke at this global level that would deploy axiomatic systems without itself being a part of the axiomatic systems in question, as if there were a super-State governing these deployments—the very position or role that Deleuze and Guattari's account of capitalism (as a system of decoding) begins by undoing. Nonetheless, the citation above seems to point in this direction when it invokes capital as possessing a general power of definition—"defined by capital as a 'right'"—but in an extremely unclear way. This problem goes well beyond the deployment of the categories of the axiomatic, and will thus be put aside until later in the analysis, although this same passage also indicates in passing what will provide the means of recomposing in a more consistent fashion Deleuze and Guattari's position, namely the market.

PUISSANCE

Deleuze and Guattari's fourth point falls under the heading "Power [*Puissance*]." It is here that the gap between mathematics and their political analysis begins to yawn more widely. It is also here that the mathematical sense of the problematic in question is presented in the most impoverished, not to say erroneous, fashion.

In set theory, let's recall, power means *size* and not *capacity*; thus $\mathbb{N}$ has the same power as $\mathbb{Q}$ or $\mathbb{Z}$, but is weaker in power than $\mathbb{R}$. In the simplest case, as Blanché notes, "two sets are said to have the same *power* [*puissance*] when a bi-univocal correspondence can be established between their elements (that is, when every element of one corresponds to one and only one element of the other, and vice versa)." (A 88n1) The word 'power' in the axiom of the Power Set is also to be understood in this sense, since the power set is always larger in size than the set on which it is based.[22]

What is specifically at issue in Deleuze and Guattari, and in the text from Blanché to which they refer, is the power of the continuum. The note just cited continues:

> Recall that [...] for finite sets, to have the same power comes down to having the same *number* of elements; for infinite sets, the weakest power is that of the *denumerable* (the indefinite series of the natural numbers); the power of the *continuum* [*la puissance du* continu] (that, for example, of the points in the line, or the set of real numbers) is superior to that of the denumerable; and finally, a set whose power surpasses that of any set whatever can always be constructed. (A 88n1)

When we turn to Deleuze and Guattari, what is most striking is that their extrapolation threatens to confuse the respective questions of size and capacity. The key moments of their discussion for our purposes here, found in the context of a remarkable analysis of war, are as follows:

> Let us suppose that the axiomatic necessarily marshals a power higher than the one it treats, in other words, than that of the sets serving as its models. This is like a power of the continuum, tied to the axiomatic but exceeding it. We immediately recognize this power as a power of destruction, of war, a power incarnated in financial, industrial, and military technological complexes that are in continuity with one another [...] There is a continuous 'threshold' of power that accompanies in every instance the shifting of the axiomatic's limits; it is as though the power of war always supersaturated the system's saturation, and was its necessary condition. (TP 466, translation modified)[23]

There are many peculiar things in play here. It is strange to see, first of all, two analogical formulations that trouble any simple literality: "this is *like* the power of the continuum," and "it is *as though* the power of war." (TP 466, emphasis added) But the more important question concerns the higher power invoked in the first sentence. Deleuze and Guattari are making reference here to a famous problem in the history of axiomatic set theory, discovered by Thoralf Skolem and first presented in 1922. In brief, what is known as Skolem's 'paradox' (it is not strictly speaking paradoxical) turns on the potential for a disparity in size between the sets constituting a model of an axiomatic system and the set of the axioms themselves.[24] Consider, for example, the fact that ZF has a denumerable number of models (ie., it is of the same size as $\mathbb{N}$), but that these model sets, such as $P(\mathbb{N})$ that are non-denumerable. This kind of situation led Skolem to conclude that "set-theoretic notions are relative,"[25] meaning that the conclusions about the nature of any set under consideration will differ depending on the axiomatic system brought to bear—in particular, as Blanché notes, "the power [*puissance*] of a set is relative to the axiomatic deployed." (A 89) While this might seem a detour from Deleuze and Guattari's analysis, in fact it cuts to the heart of not just their use of the notion of *puissance*, but the claims that come after it which concern, precisely, denumerability, a notion around which they hope to make turn an account of revolutionary politics.

If we return to the first moment of the text on *puissance*, we see immediately that Skolem's paradox is being invoked, and Deleuze and Guattari are clearly referencing Blanché's analysis.[26] However, the second sentence gives pause (for more than its use of analogy): "Let us suppose that the axiomatic necessarily marshals a power higher than the one it treats, in other words, than that of the sets serving as its models. This is like a power of the continuum, tied to the axiomatic but exceeding it." This seems to confuse the two senses of 'continuum' noted above. On the one hand, it is the continuum hypothesis that exceeds at least ZF in its basic form.[27] On the other, the upshot of Skolem's paradox is not that there are sets whose powers exceed the reach of the axioms, but rather that the axioms can always assert the existence of sets of greater power than the power of the model itself.

Perhaps the reason for Deleuze and Guattari's confusion on this point is the manner in which Blanché presents Skolem's observation. Rather than starting with the axiomatic and playing a denumerable model and a licensed non-denumerable set off of one another, he starts with the non-denumerable set and its inability to be axiomatised absolutely (ie., in a way that is not relative) due to the existence of a denumerable model. Thus, "[t]he continuum [...] cannot be axiomatically conceived in its structural specificity, since every axiomatic that can be given of it will entail a denumerable model." (A 88) If we start with the continuum in this way, as what exceeds axiomatisation, it is a small step to conceive—however incorrectly—of Blanché's point as pertaining to the continuum hypothesis, which does indeed exceed the axiomatisation of ZF, at least as it stands (that is, without the supplementation of a further axiom or axioms).[28]

If we turn from this first confusion to the question of the sense of *puissance*, a further problem presents itself. The text begins, let's recall, with these words:

> Let us suppose that the axiomatic necessarily marshals a power higher than the one it treats, in other words, than that of the sets serving as its models. This is like a power of the continuum, tied to the axiomatic but exceeding it. We immediately recognize this power as a power of destruction, of war, a power incarnated in financial, industrial, and military technological complexes that are in continuity with one another. (TP 466)

As we have already noted, though, the literal sense of power in set theory is *size*. Deleuze and Guattari clearly though have capacity in mind here, and thus it is difficult to see how it is possible to "immediately recognize" in this static and quantitative determination anything resembling destruction and war, no matter the radicality with which the latter is defined.

THE INCLUDED MIDDLE

The drift from the literal logico-mathematical sense of the axiomatic framework becomes even more pronounced when Deleuze and Guattari turn their attention to the figure of the included middle or included third [*le tiers inclu*]. While the logico-mathematical provenance of this reference is not made clear in their discussion, there are some obvious touchpoints, the first being the classical logical notion of the excluded middle. Blanché's presentation in passing of the law of the excluded middle follows the classical form, invoking two other fundamental logical laws first expounded (if not quite in this form, and not with unqualified approval) by Aristotle:

> Of two contradictory propositions *p* and *not-p*, the principle of contradiction teaches that they cannot both be true: at least one is false. For a long time, this principle has been associated with that of the excluded middle, which

> states that such propositions cannot both be false: at least one is true. The conjunction of these two principles gives what is called the principle of bivalence [*le principe de l'alternative*]: of two such propositions, one is true and the other false. (A 50)[29]

This classical framework was rejected in the work of intuitionist logicians, first and foremost LEJ Brouwer. Deleuze and Guattari introduce this point in the following note, appended to an earlier discussion of the heteromorphy of States:

> The 'intuitionist' school (Brouwer, Heyting, Griss, Bouligand, etc.) is of great importance in mathematics, not because it asserted the irreducible rights of intuition, or even because it elaborated a very novel constructivism, but because it developed a conception of *problems*, and of a *calculus of problems* that intrinsically rivals axiomatics and proceeds by other rules (notably with regard to the excluded middle). (TP 570n61)

Two points need to be made here. The first is that nothing about either intutionism itself, nor the absence of the law of the excluded middle from intuitionist logic is necessarily at odds with axiomatisation.[30] In fact, there have been many axiomatisations of intuitionist thought in a variety of areas, most notably (given the current context) an intuitionist formulation of ZF set theory known as IZF.[31] However, it is certainly the case that the law of the excluded middle (LEM) was one of Brouwer's first targets. He famously argues that certain basic proofs in classical analysis quickly come to ruin when the role of LEM in their justification is examined.[32] More generally, Brouwer's problem with LEM is that it makes possible assertions that can under no circumstances be subject to "any empirical corroboration,"[33] meaning that, whatever its intuitive appeal, it constitutes what Errett Bishop called a "principle of omniscience."[34]

The second point is that the rejection of LEM does not necessitate the truth of a law of the included middle (which would be a truly perverse use of the law of bivalence), nor do any intuitionist mathematicians make such a claim. The trespass of LEM, whether in the direction of a third option between true and false (trivalent logic),[35] or by insisting that there is an irreducible vagueness attached to determinations of the truth of propositions—an idea Deleuze and Guattari gesture towards at various points in "Apparatus of Capture" under the title of fuzzy logic—is nonetheless an open possibility.

So far, we have seen two problems with Deleuze and Guattari's account on this point: it wrongly suggests that intuitionism is hostile to axiomatisation, and it assumes that the rejection of LEM necessarily entails something like a law of the included middle. If we turn to the substance of the argument in *A Thousand Plateaus*, however, a third and even more serious problem emerges: the mathematical or logical material we have just invoked plays no role at all in their formulation. In fact, the passage in question begins with the following words: "No one has demonstrated more convincingly than Braudel that the capitalist axiomatic requires a center and that this center was constituted in the North, at the outcome of a long historical process." (TP 468) The rest of the text rests on this—geo-political—notion of the center, and alongside Braudel, we find no mathematicians, but rather Samir Amin and Antonio Negri. Consequently, it is difficult to avoid the conclusion that the use of the notion of the included middle is nothing but metaphorical in nature.

NON-DENUMERABILITY

A similar fate appears to befall the final two topics that make up the survey of the capitalist axiomatic, those appended to the concepts of minority and undecideable propositions respectively.

Deleuze and Guattari's basic definition of a minority in this context is as a non-denumerable set. According to set theory, as we have seen, a non-denumerable set is any infinite set whose cardinality, power or size is larger than that of $\mathbb{N}$, such as $P(\mathbb{N})$. However, when they come to specify what this means in the context of the political analysis, they will make a number of claims that are strictly incompatible with the set-theoretic framework they are drawing upon, two of which are particularly problematic.

The first is as follows: "A minority can be small in number; but it can also be the largest in number, constitute an absolute, indefinite majority [...] A minority can be numerous, or even infinite." (TP 469) If we read here minority as non-denumerable set (as Deleuze and Guattari wish), it is clear that very little of this is based in set theory. We are presented here with a scale that runs *small in number—numerous, even infinite—largest in number, absolute*.

However, a non-denumerable set cannot be small, or even numerous, since all such determinations concern denumerable sets. Nor can it be simply infinite, since the first rank of infinite sets like $\mathbb{N}$ are denumerable. Moreover, a fundamental claim in set theory generally conceived is there is no absolute, no 'largest' set. This is the upshot of the axiom of the power set, for starters, since it asserts the existence of a set larger in size than any given set—a basic insight that is known as Cantor's theorem. Consequently, it is strictly untrue (for both cases) that "What distinguishes them is that in the case of a majority the relation internal to the number constitutes a set that may be finite or infinite, but is always denumerable, whereas the minority is defined as a non-denumerable set, however many elements it may have." (TP 470)

Second, they will assert that what distinguishes minority (non-denumerable) from majority (denumerable) is not a difference in degree but a difference in kind:

> What characterizes the non-denumerable is neither the set nor its elements; rather, it is the *connection*, the 'and' produced between elements, between sets, and which belongs to neither, which eludes them and constitutes a line of flight. The axiomatic manipulates only denumerable sets, even infinite ones, whereas the minority constitute 'fuzzy', non-denumerable, non-axiomatisable sets, in short, 'masses', multiplicities of escape and flux. (TP 470)

Let′s put aside the basic point that a set is nothing other than its members, a thesis that Deleuze and Guattari clearly overlook here. The crucial problem is that there is no possible distinction in kind, in set theory, between different ways of belonging, between (for example) connection and conjunction. For set theory, it is strictly size that matters and nothing else. It is concerned strictly with what, from *Bergsonism* onwards, Deleuze will define as extensive multiplicities; there is no room at all for intensive multiplicity in set theory. As a result, one must either proceed without reference to set theory if the notion of intensive multiple is to be retained (the decision Deleuze explicitly makes in his work elsewhere), or dispense with the category of intensive multiplicity. In this text, though, it appears that some mixture of the two is being advanced.

All of this also pertains to the notion, invoked earlier in the discussion of minority, that "what defines a minority is, then, not the number but the relations internal to the number." (TP 469) Set theory though only insists on one such kind of relation, the relation of belonging, and this relation holds both for finite, infinite denumerable and infinite non-denumerable sets. The moment a *second* relation is introduced, at least one that would not be reducible to belonging as the primitive form (as is the case with the relation of equality, or the set-theoretic relation of inclusion), we are no longer dealing with set theory, regardless of the degree to which it is reoriented by concerns that arise on the basis of intuitionist and constructivist concerns.[36]

THE UNDECIDABLE

Given the necessary disjunction between the notions of power as size and power as capacity that lead to our concern with the analysis of war, one might feel warranted in thinking that the very final section of the plateau, which begins with the following words, might already be left aside on the grounds that it departs from the mathematical framework: "It will be objected that the axiomatic itself marshals the power of a non-denumerable infinite set: precisely that of the war machine." (TP 471) It is surely difficult to see in the concept of the war machine anything to which we might give the name 'non-denumerable set'. However, the analysis proceeds under the heading of the undecideable, a famous and fundamental *topos* in modern logic, and as such is worth considering.

Deleuze and Guattari invoke the undecideable in order to address the incapacity, at the level of capitalist bureaucracy, of coming to grips with the possibilities of revolution that insist at the fringes of the axiomatic. These possibilities, on their view, are at once produced by the dynamic movement of capitalism itself (always expanding its limits), while lying beyond the reach of the axiomatic and belonging strictly speaking to a group-subject of which they do not hesitate to say that it "finds its figure or its universal consciousness in the proletariat." (TP 472) Undecideable propositions are, on this terrain, problems produced by the movement of capital that, insofar as they are unable to be resolved by it, provide opportunities for alternative modes of relation that Deleuze and Guattari will unite under the heading of revolutionary *connection*, as opposed to the capitalist *conjunction* of decoded flows effected by the axioms.

Here again, Deleuze and Guattari invoke here Blanché, and his treatment of the undecideable, which he introduces under the heading 'The Limits of Demonstrations of Non-Contradiction', and in the context of an account of the extent to which the axiomatic method can succeed in grounding a system of propositions (axioms, in our case) in its entirety. He notes that this ambition came to ground against an entirely unexpected and striking pair of results, presented by Gödel in 1931. Blanché's lucid little summary text is emblematic of the clarity of his writing:

> In two famous theorems of metamathematics, he established, first, that a non-contradictory arithmetic cannot constitute a complete system, and necessarily includes undecideable statements; and, second, that the affirmation of the non-contradiction of the system itself figures among these undecideable statements. (A 68)

In other words, the first result is that, in a formal system of complexity sufficient to include arithmetic (for example ZFC), there will always be propositions that cannot be proven or disproven within the system. The second result is that no formal system is capable of proving its own consistency, due to the fact that such a proof has no means of accounting for these undecideable propositions themselves. An apparent resolution of this second limit would be to formalize the first-order system within a higher-order system—for example, in the way that PA can be formalized within ZFC. Clearly, though, this latter system is prey to the same results, and the response ultimately becomes either to give way to an infinite regress, or accept this irreducible fact about such formal systems.

If this latter point is worth mentioning in this context, it is because the structure of the point recapitulates at the level of mathematical logic the basic structure of the dynamism of capitalism on Deleuze and Guattari's view—that is, the ever-displaced limit of what is decoded on the one hand and axiomatised on the other—and this is what appears to be at work in their deployement of the idea. But, as we have noted a number of times already, the static world of mathematical logic and the dynamic movement of capitalism are uneasy bedfellows.

The two Gödel results have been among the most generally deployed insights from within formal science, but we would do well to note the specificity of the conditions in which they arise—formal systems in which one can formulate recursive arithmetic. For this reason, the many popular extrapolations of these results (of which Hofstadter's *Gödel, Escher, Bach* is perhaps the most famous case) which tend to emphasise some kind of vague but fundamental incompletion at the level of discourse, knowledge or human cognition, make too little of Gödel's work by making too much of it. It is notable, in this context, that when Derrida invokes Gödel's notion of undecideability, he is careful to insist that it is *analogical* to his notion of *différance*, and not "literally the same."[37] Unfortunately, it is this latter claim that once more troubles our assent to Deleuze and Guattari's analysis, which has nothing at all to do with formalized arithmetic—here, neither literally nor 'metaphorically', even though it is difficult to see what this latter would amount to.

CONCLUSION. BEYOND THE LITERAL

In sum, Deleuze and Guattari's use of set theory involves certain erroneous presentations of the axiomatic method and threatens others. Moreover, the two accounts are not the same "in a literal sense." (TP 461) One of the problems here is certainly the insistence on the category of the literal. Deleuze's antipathy to metaphor is well-known, but a deposition of the category of the literal would seem to necessarily follow, since the two both rely upon the same semiotic arrangement. Elsewhere, it is clear that they understand this point—hence the invocation of the fundamental role of variation in the "Postulates of Linguistics" plateau.

The point of this analysis was not to show that Deleuze and Guattari are simply wrong, or to engage in Sokal-style crypto-moralising, but rather to simply outline the dramatic gap between the account they say they are offering and the one they in fact deploy. Consequently, it would be simplicity itself to put to one side this analysis on the grounds that what really matters is not the initial inspiration for the concepts in question, nor the means, regulated or otherwise, of extrapolating these concepts from the (mathematico-logical) axiomatic, but rather the concepts themselves, the framework they provide us with to engage with contemporary reality.

This could arguably be achieved by dropping the more detailed parallels established with set theory. After all, the concept of the axiom itself does not belong solely to that domain, nor even in the final analysis to mathematics itself. One could instead begin with the minimal definition that Deleuze and Guattari themselves provide: that the axiom is a rule that deals "directly with purely functional elements and relations whose nature is not specified, and which are immediately realized in highly varied domains simultaneously." (TP 454) From here, it would be a matter of investigating the nature and variety of axioms in this sense, their conjoint and disjunct functioning, the specific logic of their combinations. This would be, not a literal mathematical account, but a *philosophical* theory of the capitalist axiomatic would result. Such an account would have to rely upon the authority of the concept alone—but it is in Deleuze's work that we find one of the best arguments for the capacity of the concept to meet the challenges of our time. And indeed, this is the kind of argument that Deleuze and Guattari will go on to offer in *What is Philosophy?*, even if they retain there a certain false image of set theoretic mathematics.

NOTES

1. The recent work of Knox Peden has turns around aspects of this tradition. See, on the one hand, the first three chapters of his *Spinoza Contra Phenomenology* (Stanford: Stanford University Press, 2014), which deal with Jean Cavaillès, Ferdinand Alquié and the more recent work of Jean-Toussaint Desanti. Along with Peter Hallward, Peden is also the editor of the two volume work on the important journal *Cahier pour l'analyse*, *Concept and Form* (New York: Verso, 2012) in which a particularly stringent form of this philosophical attention unfolded.
2. The work of Simon Duffy has been decisive in unfolding these mathematical resources of Deleuze's thought. See in particular *Deleuze and the History of Mathematics* (London: Bloomsbury, 2013) and *The Logic of Expression: Quality, Quantity and Intensity in Spinoza, Hegel and Deleuze* (Durham: Ashgate, 2006).
3. Gilles Deleuze and Félix Guattari, *What is Philosophy?*, trans. Hugh Tomlinson and Graham Burchell (New York: Columbia University Press. 1994), 120. Hereafter referred to in text as WP.
4. This definition of the size of power sets also holds for finite power sets: for example, the power set of a set A = {a, b, c} will have eight members, such that P(A) = {a, b, c, {a, b}, {a, c}, {b, c}, {a, b, c}, ø}. The latter two members are known as the maximal subset and the null-set, the set without members.
5. Gottfried Frege, "Letter to Russell," in *From Frege to Gödel: A Sourcebook in Mathematical Logic, 1879-1931*, ed. Jean van Heijenoort (Cambridge, MA: Harvard University Press, 1967), 127.
6. In fact, all of these paradoxes, which emerged after Russell's initial engagement with Cantor, and then Peano after that, were authored by Russell himself. Even the famed Burali-Forti paradox was concocted by Russell after reading "A question on transfinite numbers" by the former logician, who in this piece explicitly claims to have overcome the problem that he had uncovered through a *reductio ad absurdum*. More recent versions of this form of paradox, which essentially involve (like its ancient predecessor, the liar's paradox) self-referentiality, have been developed, notably Curry's paradox (which is significantly more consequential that any of the above). What is interesting to note is the fact that very few paradoxes produced thus far in any field of mathematical logic are provably other than paradoxes of self-reference. Yablo's paradox, perhaps the most famous example of an apparently non-self-referential paradox, has recently been formulated in such a way to indicate the contrary. For some discussion of Yablo and more generally self-reference in paradox, see T.B. Jongeling, T. Koetsier and E. Wattel, "Self-Reference in Finite and Infinite Paradoxes," *Logique & Analyse* 177–178 (2002), 29-46.
7. I pass over here the fact that two of these (viz., Separation and Replacement) are axiom schemata rather than axioms *per se*, that is, they govern the deployment of an infinite number of primary axioms. I also pass over the fact that the axiom of Choice is still considered in some quarters to be controversial, and is thus either left out or indicated by describing the axiom set as ZFC, Zermelo-Frankel axiomatic set theory with the axiom of Choice.
8. This note, beyond its peculiar rendering of Badiou's project has its own problems, including but not limited to the following: the state of a situation is not a subset, nor is the state itself represented (it is itself the order of representation), nor is it itself indiscernible; the elements of the evental site do not depend on the generic procedures of art, science, politics and love; the event is not made, nor is there any conceivable power that might be able to do this in Badiou's system.

Nevertheless, the note on Badiou in *What is Philosophy?* is not without merit. For a discussion of the forceful critique of Badiou's account of conditioning in thought that it implies, see Jon Roffe, "Deleuze's Badiou," in *Badiou and his Interlocutors: Lectures, Interviews and Responses*, ed. AJ

Bartlett (forthcoming 2017, Bloomsbury).
9. See for example Gilles Deleuze, *Difference and Repetition*, trans. Paul Patton (New York: Columbia University Press, 1995), 172.
10. Here, they are close (at least in a certain respect) to Badiou, whose recourse to mathematics in accounting for contemporary reality is well known—see not only the (troubling, in my view, or at least unjustified) linking of the axiom of the power set with the figure of the State in a political sense in *Being and Event*, trans. Oliver Feltham (London: Continuum, 2005), 95, but also the critique of the role of number in capitalism with which he opens *Number and Numbers*, trans. Robin Mackay (Cambridge, MA.: Polity Press, 2008), 1-4.
11. Gilles Deleuze and Félix Guattari, *A Thousand Plateaus*, trans. Brian Massumi (London: Athlone, 1988), 461. Hereafter referred to in text as TP.
12. Gilles Deleuze and Félix Guattari, *Anti-Oedipus: Capitalism and Schizophrenia*, trans. Robert Hurley, Mark Seem and Helen R. Lane (London: Athlone, 1984), 139.
13. The generalization of this analysis, pursued in chapter six of Jon Roffe, *Abstract Market Theory* (Basingstoke: Palgrave, 2015), allows for a partial correction of some of the problems that emerge in Deleuze and Guattari's use of set theory treated here.
14. Roberte Blanché, *L'axiomatique* (Paris: PUF, 1955). Hereafter referred to in text as A.
15. See for example TP 554n21 and 570n61. In a fine piece, Daniel Smith has analysed the respective roles of (axiomatic) state mathematics and (problematic) nomad science. See Daniel W. Smith, "Badiou and Deleuze on the ontology of mathematics," in *Think Again*, ed. Peter Hallward (London: Continuum, 2004), 77-93.
16. For Blanché's discussion of PA, see A 41-2.
17. The other four are derivative on these five, and turn around the use of the second, third and fifth axioms to define relative size.
18. This text, drawn from the third volume of Marx's *Capital*, is cited at Deleuze and Guattari 1984, p. 231.
19. Deleuze and Guattari, *Anti-Oedipus*, 246.
20. Deleuze and Guattari, *What is Philosophy?*, 102.
21. See TP 435 for the logic of this point. Deleuze and Guattari insist on a further logically prior displacement also, when they write that "It is not the State that presupposes a mode of production; quite the opposite, it is the State that makes production a 'mode.'" (TP 429)
22. It is worth noting in passing that, in French, this axiom does not include the word *puissance*, but is rather called *l'axiome de l'ensemble des parties*, the subset axiom.
23. The original reads: "Supposons que l'axiomatique dégage nécessairement une puissance supérieure à celle qu'elle traite, c'est-à-dire à celle des ensembles qui lui servent de modèles." (Deleuze and Guattari, 1980, *Mille Plateaux,* Paris: Editions de Minuit, 1980, 582). Somewhat inexplicably, given Deleuze and Guattari's insistence on the literal character of their use of set theory, Brian Massumi renders *ensemble* in different ways, often obscuring (as when he renders it, as he does in this passage, as 'aggregate') the technical use of the term.
24. The first presentation of this result is in Thoraf Skolem, "Some remarks on axiomatised set theory," in *From Frege to Gödel*, 290-301. See also von Neumann's famous engagement with this issue, one that Blanché also invokes as we will see, in John Von Neumann, "An axiomatisation of set theory," in *From Frege to Gödel*, esp. 412-3 (§4). More recently, in his "Models and Reality. *The Journal of Symbolic Logic* 45:3 (1980), 464-82, Hilary Putnam has finessed the problem from the side of the philosophy of language, drawing out its consequences for what he calls moderate realism. His (deflationary) conclusion turns on a concomitant relativisation of the notion of model to its

concrete uses (the key paragraph is found at the bottom of 481). For a more substantial, indeed thorough-going, critique of the problematic equivocation that gives this 'paradox' its force, see Timothy Bays' excellent "Reflections on Skolem's paradox," http://www3.nd.edu/~tbays/papers/pthesis.pdf, accessed 19 February 2014. Bays effectively shows that the apparently paradoxical nature of the problem arises when we confuse the set-theoretic and propositional levels of the analysis—the kind of point that is perfectly coherent with what Deleuze will say about the relationship between problem and proposition in *Difference and Repetition* and *The Logic of Sense*.
25. Skolem, "Some Remarks," 300.
26. One need only compare the text cited above, the main note referring to Blanché's book (TP 570n54), and the passage at A 88-9.
27. Note that one (positive) resolution of the continuum hypothesis, proposed by Kurt Gödel though not particularly well-regarded by mathematicians (nor by Gödel himself in the end), involves the addition of the axiom of constructibility. Badiou, following Gödel, affiliates with Leibnizian metaphysics, see Badiou, *Being and Event*, 295-23.
28. For an excellent and philosophically rich investigation into what the assertion of new axioms might mean in the context of ZFC and in light of Gödel's results, see Solomon Feferman, "Does mathematics need new axioms?," *American Mathematical Monthly* 106 (1999), 99-111. More generally on this issue, see the discussion in Penelope Maddy *Realism in Mathematics* (Oxford: Clarendon Press, 1989), 107-42.
29. This latter is what Brouwer calls "the *principle of the reciprocity of the complementary species*, that is, the principle that for every system the correctness of a property follows from the impossibility of the impossibility of this property." (L.E.J. Brouwer, "On the significance of the principle of the excluded middle in mathematics, especially in function theory," in *From Frege to Gödel*, 335.
30. Brouwer here constitutes a partial exception, since he rejects the project of axiomatisation itself.
31. While first presented in the 1970s, a common contemporary point of reference is the work of Peter Aczel and Michael Rathjen (see for example Aczel and Rathjen. *Notes on Constructive Set Theory*. http://www.maths.manchester.ac.uk/logic/mathlogaps/workshop/CST-book-June-08.pdf. Accessed 10 September 2014, §3.2.
32. His two main examples are the claims "The points of the continuum form an ordered point species," and "Every mathematical species is either finite or infinite." Brouwer's arguments on these matters have been challenged from a number of points of view; the value in bringing them up here is only to illuminate the point that Deleuze and Guattari are engaging with.
33. Brouwer, "The principle of the excluded middle," 336.
34. Errol Bishop, *Foundations of Constructive Analysis* (New York: McGraw-Hill Book Company, 1967), 19. Conversely, it is often asserted that the intutionist and constructivist approaches have as their goal the rejection of any non-computational method of proof.
35. Or indeed any number of truth-values. On this, see Emil Post, "Introduction to a general theory of elementary propositions," in J. van Jeijenoort (Ed.), *From Frege to Gödel: A Sourcebook in Mathematical Logic, 1879-1931*. (Cambridge, MA: Harvard University Press, 1967), especially §11 (279-81).
36. All of these points also hold for the following subsequent programmatic assertion: "*At the same time as capitalism is effectuated in the denumerable sets serving as its models, it necessarily constitutes non-denumerable sets that cut across and disrupt these models*." (TP 472)
37. "Allusion, or 'suggestion' as Mallarmé says elsewhere, is indeed that operation we are here *by analogy* calling undecidable. An undecidable proposition, as Gödel demonstrated in 1931, is a proposition which, given a system of axioms governing a multiplicity, is neither an analytical nor

deductive consequence of those axioms, nor in contradiction with them, neither true nor false with respect to those axioms. *Tertium datur*, without synthesis." (Jacques Derrida, *Dissemination*, trans. Barbara Johnson (Chicago: University of Chicago Press, 1970), 219, emphasis added.